Anualidades Anticipadas

Rpta ① Demuestre que:

a) $$\left[\frac{(1+i)^{n+1}-1}{i}\right]-1=(1+i)\left[\frac{(1+i)^{n}-1}{i}\right]$$

Despejamos: $$=(1+i)\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$=\left[\frac{(1+i)^{n}-1}{i}\right]+\cancel{i}\left[\frac{(1+i)^{n}-1}{\cancel{i}}\right]$$

$$=\left[\frac{(1+i)^{n}-1}{i}\right]+(1+i)^{n}-1$$

$$=\frac{\left[(1+i)^{n}-1\right]+i(1+i)^{n}}{i}-1$$

$$=\left[\frac{(1+i)^{n}+i(1+i)^{n}-1}{i}\right]-1$$

$$=\left[\frac{(1+i)^{n}(1+i)-1}{i}\right]-1$$

$$=\left[\frac{(1+i)^{n+1}-1}{i}\right]-1$$

Queda demostrado la igualdad.

b) $$\frac{1-(1+i)^{-(n-1)}}{i} + 1 = (1+i)\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Despejamos: $$= (1+i)\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + \cancel{i}\left[\frac{(1+i)^n - 1}{\cancel{i}(1+i)^n}\right]$$

$$= \frac{(1+i)^n - 1}{i(1+i)^n} + \frac{(1+i)^n - 1}{(1+i)^n}$$

$$= \cancel{\frac{(1+i)^n}{i(1+i)^n}} - \frac{1}{i(1+i)^n} + \cancel{\frac{(1+i)^n}{(1+i)^n}} - \frac{1}{(1+i)^n}$$

$$= \frac{1}{i} - \frac{1}{i(1+i)^n} + 1 - \frac{1}{(1+i)^n}$$

$$= \frac{1}{i} - \left[\frac{1}{i(1+i)^n} + \frac{1}{(1+i)^n}\right] + 1$$

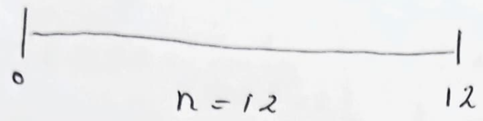
$$= \frac{1}{i} - \left[\frac{(1+i)}{i(1+i)^n}\right] + 1$$

$$= \frac{1}{i} - \frac{(1+i)^{1-n}}{i} + 1$$

$$= \left[\frac{1-(1+i)^{-(n-1)}}{i}\right] + 1$$

– Queda demostrado la igualdad.

Rpta (3)



$i = 0.03$

- Factor de Recuperación de Capital (FRC).

Como: $\boxed{FRC = \dfrac{i(1+i)^n}{(1+i)^n - 1}}$; $\boxed{Ra = \dfrac{P}{(1+i)} \times FRC}$

Reemplazamos y hacemos que $P = 1$

$$Ra = \frac{1}{(1+0.03)}\left[\frac{(0.03)(1+0.03)^{12}}{(1+0.03)^{12} - 1}\right]$$

$$Ra = \frac{1}{1.03}\left[\frac{0.0427728266}{0.4257608868}\right]$$

$$Ra = 0.09753600533$$

- Factor de Capitalización la serie (FCS).

Como: $\boxed{FCS = \dfrac{(1+i)^n - 1}{i}}$; $\boxed{S = Ra\,(1+i) \times FCS}$

Reemplazamos y hacemos que $Ra = 1$

$$S = (1)(1+0.03)\left[\frac{(1+0.03)^{12} - 1}{0.03}\right]$$

$$S = 1.03\left[\frac{0.4257608868}{0.03}\right]$$

$$S = 14.61779045$$

- Factor de Deposito al fondo de amortización (FDFA).

Como: $\boxed{FDFA = \frac{i}{(1+i)^n - 1}}$; $\boxed{Ra = \frac{S}{(1+i)}\left[\frac{i}{(1+i)^n - 1}\right]}$

Reemplazamos y hacemos que $S = 1$

$$Ra = \frac{1}{1+0.03}\left[\frac{0.03}{(1+0.03)^{12} - 1}\right]$$

$$Ra = 0.0684097917\!\!1$$

- Factor de actualización de la serie (FAS).

Como: $\boxed{FAS = \frac{(1+i)^n - 1}{i(1+i)^n}}$; $\boxed{P = Ra(1+i)\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$

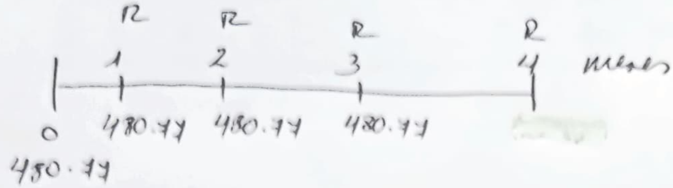
Reemplazamos y hacemos que $Ra = 1$

$$P = 1(1+0.03)\left[\frac{(1+i)^{12} - 1}{0.03(1+0.03)^{12}}\right]$$

$$P = 1.03\left[\frac{0.4257608868}{0.0427728266\,1}\right]$$

$$P = 10.25262411$$

Rpta (5)



TEM=0.04 Como: $\boxed{R = Ra(1+i)}$

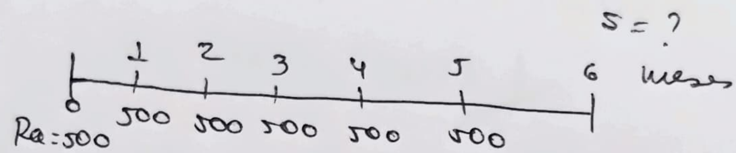
Reemplazamos:

$$R = (480.77)(1+0.04)$$

$$R = 500$$

- La restitución equivalente es R = 500 um.

Rpta (7)



TEM: 0.03 Como: $\boxed{S = \frac{Ra(1+i)[(1+i)^n - 1]}{i}}$

Reemplazamos:

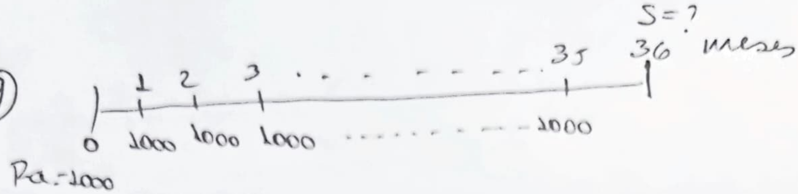
$$S = \frac{(500)(1+0.03)[(1+0.03)^6 - 1]}{0.03}$$

$$S = \frac{(500)(1.03)(99.93693271)}{0.03}$$

$$S = 3331.23109$$

- Al término del sexto mes acumuló S = 3331.23 um.

Rpta (9)



TNA = 0.24 Cap. Mensualmente

$\div 12$

→ TEM = 0.02

Como: $\boxed{S = Ra(1+i)\left[\frac{(1+i)^n - 1}{i}\right]}$

Reemplazamos:

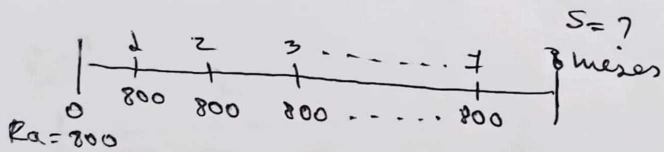
$$S = (1000)(1+0.02)\left[\frac{(1+0.02)^{36} - 1}{0.02}\right]$$

$$S = (1000)(1.02)(51.99436719)$$

$$S = 53034.25453$$

- Se puede acumular un monto de S = 53034.25 um.

Rpta (11)



TEA = 0.12

→ $TEM = (1+0.12)^{1/12} - 1$

TEM = 0.009488792935

Como: $\boxed{S = Ra(1+i)\left[\frac{(1+i)^n - 1}{i}\right]}$

Reemplazamos:

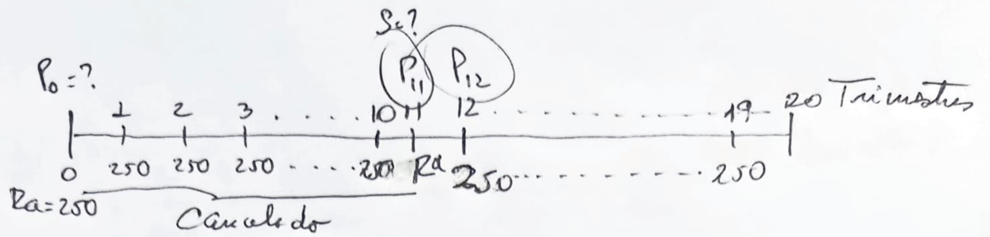
$$S = (800)(1+0.009)\left[\frac{(1+0.009)^{8} - 1}{0.009}\right]$$

$$S = (800)(1.009)(8.27078545)$$

$$S = 6679.414676$$

- Se acumuló un monto de S = 6679.41 um.

Rpta (13)



TNA = 0.36 Cap. Trimestralmente

$\div 4$

TET = 0.09

Caso: $\boxed{P = Ra(1+i)\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]}$

Reemplazando:

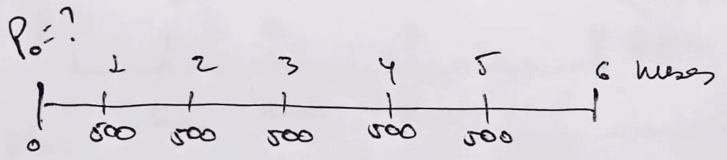
$$P_{12} = (250)(1+0.09)\left[\frac{(1+0.09)^9 - 1}{(0.09)(1+0.09)^9}\right]$$

$$P_{11} = (250)(1.09)(5.995246894)$$

$$P_{11} = 1633.704779$$

- El importe total por cancelar en esa fecha es 1633.70 um.

Rpta (15)



Ra = 500

TEM = 0.04

Caso: $\boxed{P = Ra(1+i)\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]}$

Reemplazamos:

$$P = (500)(1+0.04)\left[\frac{(1+0.04)^6 - 1}{(0.04)(1+0.04)^6}\right]$$

$$P = 2725.911166$$

- Al finalizar el préstamo se paga $n \times Ra \Rightarrow 6 \times 500 = 3000$

Hallamos el importe total del interés por pagar:

$$I = 3000 - 2725.911166$$

$$I = 274.088834$$

- El importe total del Interés por pagar es 274.09 um.

Rpta (17) $P_0 = ?$

$Ra: 2000$

$TEM = 0.015$

$TET = (1+0.015)^3 - 1$

$TET = 0.04567 8375$

Como: $P_0 = Ra(1+i)\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]$

Reemplazando:

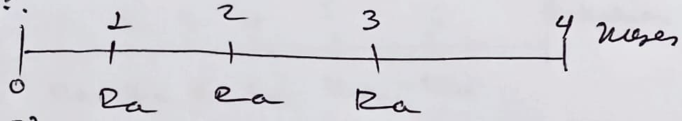
$$P_0 = (2000)(1+0.046)\left[\frac{(1+0.046)^{20} - 1}{(0.046)(1+0.046)^{20}}\right]$$

$$P_0 = (2000)(1.046)(12.93180927)$$

$$P_0 = 27045.02661$$

- El valor presente es $P_0 = 27045.03$ UM.

Rpta (19) $P = ?$ $S = 5000$



$Ra = ?$

$TEM = 0.01$

Como: $\boxed{Ra = \dfrac{S}{(1+i)}\left[\dfrac{i}{(1+i)^n - 1}\right]}$

Reemplazando:

$$Ra = \frac{(5000)}{(1+0.01)}\left[\frac{0.01}{(1+0.01)^4 - 1}\right]$$

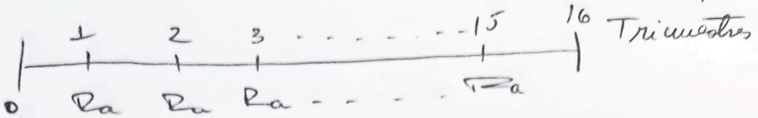
$$Ra = \frac{5000}{1.01}(0.2462810939)$$

$$Ra = 1219.213336$$

- Se deberá depositar cada 30 días $Ra = 1219.21$ UM.

Rpta (21)

$S = 20{,}000$



Ra: ?

$TEA = 0.12$

$\rightarrow TET = (1+0.12)^{3/12} - 1$

Como: $\boxed{Ra = \frac{S}{(1+i)}\left[\frac{i}{(1+i)^n - 1}\right]}$

$TET = 0.0287373 4472$

Reemplazando:

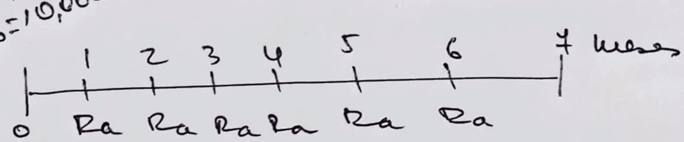
$$Ra = \frac{20{,}000}{(1+0.0287)}\left[\frac{0.0287}{(1+0.0287)^{16} - 1}\right]$$

$$Ra = \frac{20000}{1.0287}(0.051070177)$$

$$Ra = 974.145985$$

- El importe de renta constante es $Ra = 974.15$ Um.

Rpta (23) $P_0 = 10{,}000$



Ra: ?

$TEA = 0.25$

$\rightarrow TEM = (1+0.25)^{1/12} - 1$

Como: $\boxed{Ra = \frac{P}{(1+i)}\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]}$

$TEM = 0.0187692 6512$

Reemplazando:

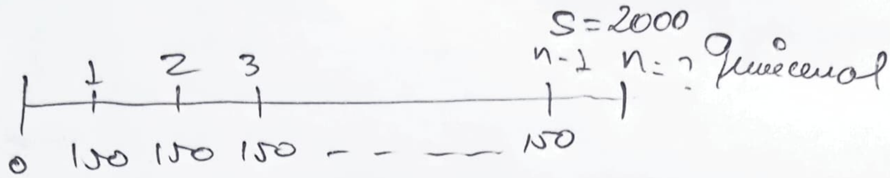
$$Ra = \frac{10000}{(1+0.0187)}\left[\frac{(0.0187)(1+0.0187)^7}{(1+0.0187)^7 - 1}\right]$$

$$Ra = \frac{10000}{1.0187}(0.1537818198)$$

$$Ra = 1509.48625$$

- El importe de las cuotas es $Ra = 1509.49$ Um.

Rpta (25)



$Ra = 150$

$TNA = 0.24 \div 12$ cap. mensual

$\hookrightarrow TEM = 0.02$

$\hookrightarrow TEQ = (1+0.02)^{15/30} - 1$

$TEQ = 0.009950493836$

Luego: $\boxed{n = \dfrac{\lg\left[\dfrac{S\,i}{Ra(1+i)} + 1\right]}{\lg(1+i)}}$

Reemplazando:

$$n = \frac{\lg\left[\dfrac{(2000)(0.00995)}{(150)(1+0.00995)} + 1\right]}{\lg(1+0.00995)}$$

$$n = \frac{(0.05260315906)}{(0.00430)}$$

$n = 12.46560798$ quincen.

- Se podrá acumular 2000 en 12.47 quincenas.

Rpta (27) $P_0 = 8500$

0	1	2	3	. . .	n-1	n = ? meses
	1650	1650	1650	. . .	1650

$Ra = 1650$

$TNA = 0.24$ Cap. Trimestralmente
$\div 4$

$TET = 0.06$

$TEM = (1+0.06)^{1/3} - 1$

$TEM = 0.01961282242$

Como: $n = \dfrac{\lg\left[1 - \dfrac{Pi}{Ra(1+i)}\right]}{\lg(1+i)}$

Reemplazando:

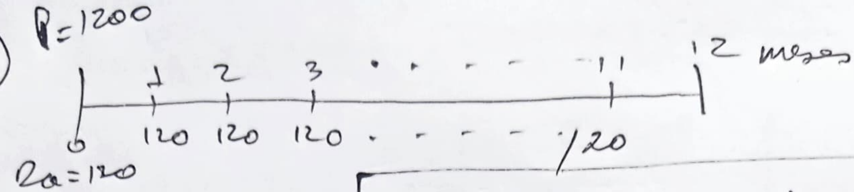
$$n = \frac{\lg\left[1 - \dfrac{(8500)(0.0196)}{(1650)(1+0.0196)}\right]}{\lg(1+0.0196)}$$

$$n = \frac{\lg(0.9009077273)}{\lg(1.0196)}$$

$$n = 5.372630521$$

- Son necesarios $n = 5.37$

Rpta (29)



TEM=? Como: $\boxed{P = Ra(1+i)\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$

Interpolamos: Como: $\boxed{\frac{a}{b} = \frac{c}{d}}$

$$\begin{array}{c|cc|c} & P & TEM & \\ a\left[\begin{array}{c}\\ \\ \end{array}\right. & 1200.186124 & 0.035 & \left.\begin{array}{c}\\ \\ \end{array}\right]c \\ & 1200 & TEM & \\ b & 1199.595987 & 0.0351 & d \end{array}$$

$$\Rightarrow \frac{1200.186124 - 1200}{1200 - 1199.595987} = \frac{0.035 - TEM}{TEM - 0.0351}$$

$$\frac{0.186124}{0.404013} = \frac{0.035 - TEM}{TEM - 0.0351}$$

$$(0.4606881462)(TEM - 0.0351) = 0.035 - TEM$$

$$(0.4606881462)TEM - 0.01617015393 = 0.035 - TEM$$

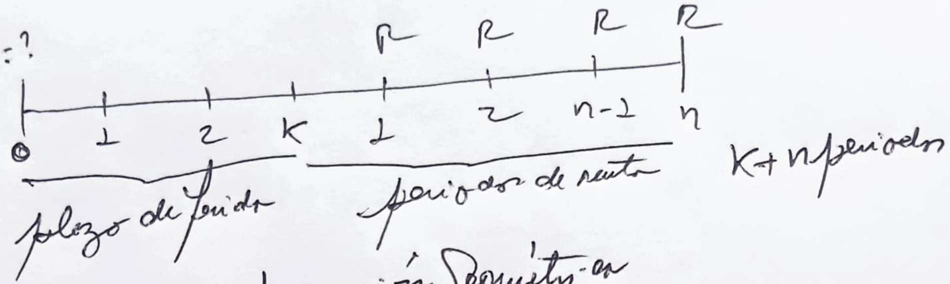
$$(1.4606881462)TEM = 0.05117015393$$

$$TEM = 0.03503153912$$

- La TEM cargada es de 3,503%.

ANUALIDADES DIFERIDAS

Rpta ① $P_0 = ?$



Como: progresión Geométrica

$$\boxed{S_n = a\cdot\frac{1-r^n}{1-r}}$$

donde: a: es el primer término de la progresión G.

r = es la razón de la progresión

n = es el número de términos

entonces:

$a = R$

$r = (1+i)^{-n}$

n: número de periodos de la renta

Para: a) una anualidad diferida vencida.

– El valor presente de la anualidad es la suma de esta progresión Geométrica $\boxed{S_n = P_0}$

$$\Rightarrow P_0 = R\cdot\frac{1-(1+i)^{-n}}{i}$$

despejamos:

$$P_0 = R\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$$

$$\boxed{P_0 = R\cdot\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]}$$

- Ahora se realiza el descuento de la anualidad diferida; multiplicandose el valor presente el factor de descuento $(1+i)^k$.

Por lo tanto: $(P_0)(1+i)^k = R \cdot \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$$\boxed{P_0 = R \cdot \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \cdot (1+i)^{-k}}$$

Para b) una anualidad diferida anticipada.

- Tomo el valor presente de una anualidad ordinaria anticipada:

$$\boxed{P_0 = Ra \cdot \frac{1-(1+i)^{-n}}{i} \cdot (1+i)}$$

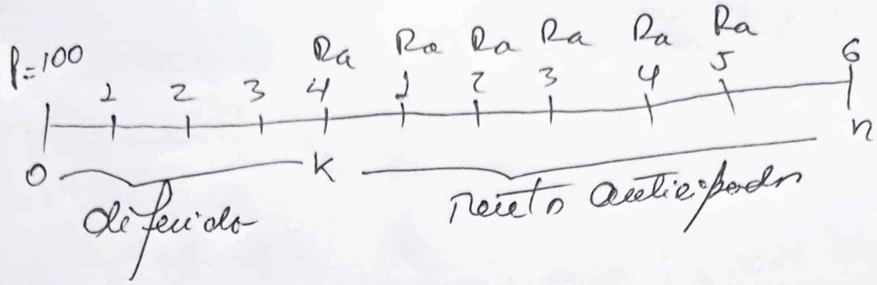
- Ahora se realiza el descuento anualidad diferido anticipado multiplicandose por el factor descuento $(1+i)^{-k+1}$.

Por lo tanto: $P_0 = Ra\left[\frac{1-(1+i)^{-n}}{i}\right] \cdot (1+i)^{-k+1}$

Se despeja: $P_0 = Ra \cdot \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \cdot (1+i)^{-k} \cdot (1+i)$

$$\boxed{P_0 = Ra \cdot \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \cdot \left[\frac{(1+i)}{(1+i)^k}\right]}$$

Rpta (3)



$i = 0.03$

Hallar: $Ra = ?$

$S = ?$

$P = ?$

Como: $$\boxed{Ra = \frac{P(1+i)^k}{(1+i)}\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]}$$

Reemplazando:

$$Ra = \frac{(100)(1+0.03)^4}{(1+0.03)}\left[\frac{(0.03)(1+0.03)^6}{(1+0.03)^6 - 1}\right]$$

$$Ra = 20.171$$

Como: $$S = Ra(1+i)\left[\frac{(1+i)^n - 1}{i}\right]$$

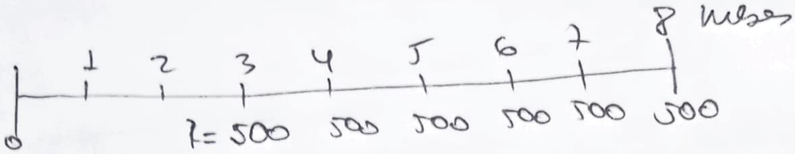
Reemplazando:

$$S = (20.171)(1+0.03)\left[\frac{(1+0.03)^6 - 1}{0.03}\right]$$

$$S = 134.389$$

Como: P es dato $\Rightarrow$ $P = 100$

Rpta (5)



TEM = 0.03

Con r: $S = R\left[\dfrac{(1+i)^n - 1}{i}\right]$

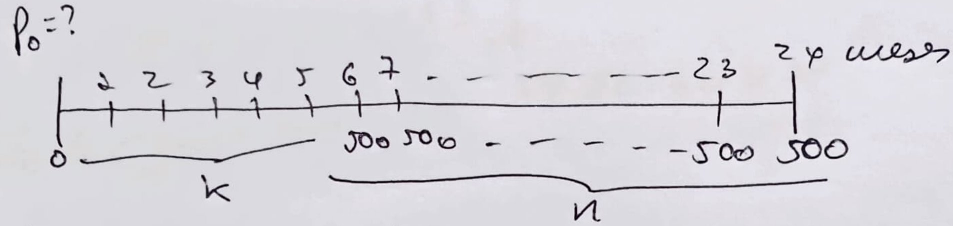
Reemplazamos:

$$S = (500)\left[\frac{(1+0.03)^6 - 1}{0.03}\right]$$

$$S = 3234.204942$$

- El monto de la anualidad → 3234.20 um.

Rpta (7)



TEM = 0.03 TEM = 0.04

Con r: $\boxed{P_0 = R\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]\left[\dfrac{1}{(1+i)^k}\right]}$

Reemplazamos:

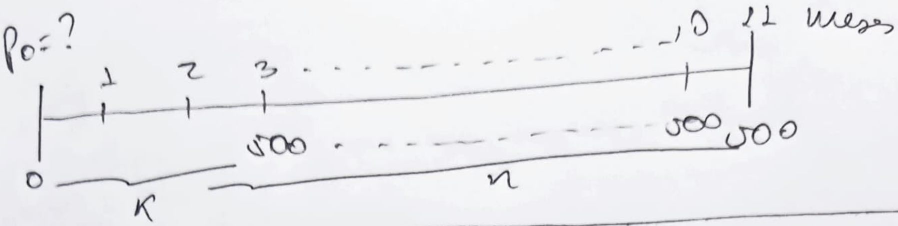
$$P_0 = (500)\left[\frac{(1+0.04)^{24} - 1}{(0.04)(1+0.04)^{24}}\right]\left[\frac{1}{(1+0.04)^5}\right]$$

$$P_0 = (500)(15.24696314)(0.8626087844)$$

$$P_0 = 6576.082171$$

- El valor presente → $P_0 = 6576.08$ um.

Rpta (9) $P_0 = ?$



TEM=0.02

Como: $\boxed{P_0 = R\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]\left[\dfrac{1}{(1+i)^K}\right]}$

Reemplazamos:

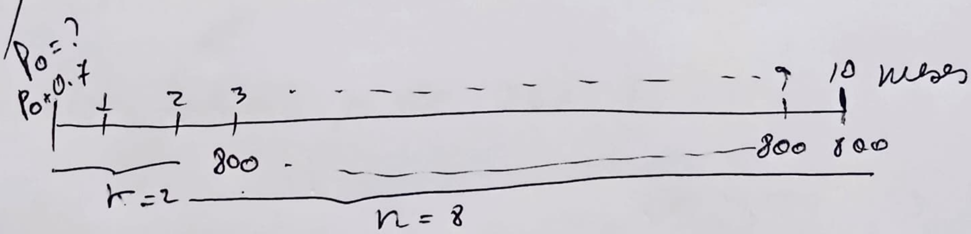
$$P_0 = (500)\left[\frac{(1+0.02)^9 - 1}{(0.02)(1+0.02)^9}\right]\left[\frac{1}{(1+0.02)^2}\right]$$

$$P_0 = (500)(8.162236706)(0.9611687812)$$

$$P_0 = 3922.643553$$

- El importe mínimo → 3922.64 um.

Rpta (11) $P_0 = ?$



TEM=0.015

Como: $P_0 = R\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]\left[\dfrac{1}{(1+i)}\right]$

Reemplazando:

$$P_{0_{\%70}} = (800)\left[\frac{(1+0.015)^8 - 1}{(0.015)(1+0.015)^8}\right]\left[\frac{1}{(1+0.015)}\right]$$

$$P_{0_{\%70}} = (800)(7.48592508)(0.9706617486)$$

$$P_{0_{\%70}} = 5813.040902$$

$$P_0 = \frac{5813.040902 \times 100}{70} = 8304.344146$$

- El precio de contado → 8304.34 um.

Rpeemplazamos :

$$8000 = R\left[\frac{(1+0.1135)^{8}-1}{(0.1135)(1+0.1135)^{8}}\right]$$

$$8000 = R(5.081576405)$$

$$R = 1574.314615$$

Reemplazamos :

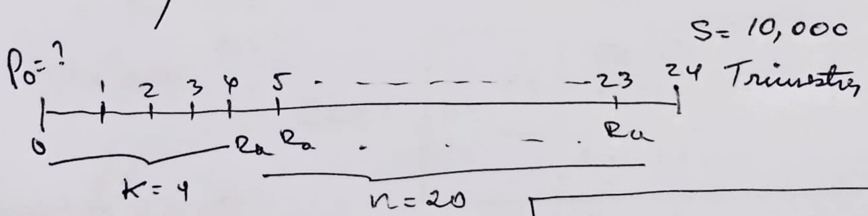
$$P_0 = \left(\frac{2}{3}R\right)\left[\frac{(1+0.092727)^{12}-1}{(0.092727)(1+0.092727)^{12}}\right]\left[\frac{1+0.092727}{(1+0.092727)^{4}}\right]$$

$$P_0 = \left(\frac{2}{3}\cdot 1574.314615\right)(7.063396583)(0.7664167323)$$

$$P_0 = 5681.72$$

- El valor presente es de 5681.72 um.

Rpta (17)



$TEA = 0.08$

$\hookrightarrow TET = (1+0.08)^{1/4}-1$

$TET = 0.01942654691$

Como: $\boxed{Ra = \frac{S}{1+i}\left[\frac{i}{(1+i)^{n}-1}\right]}$

Reemplazamos :

$$Ra = \frac{10000}{1+0.019}\left[\frac{0.019}{(1+0.019)^{20}-1}\right]$$

$$Ra = (9809.436521)(0.0413922 5388)$$

$$Ra = 406.0346869$$

- La cuota uniforme anticipada es de 406.03 um.

Rpta (19) $P_0 = 11166.33$

$R = 2000$ . . . . $R = 2000$

0 1 2 3 . . . . 8 meses

$K = 3$

TEM = 0.05

Const: $$\boxed{K = \frac{\lg\left[\frac{R}{P i}\left[1 - \frac{1}{(1+i)^n}\right]\right]}{\lg(1+i)}}$$

Reemplazando:

$$K = \frac{\lg\left[\frac{2000}{(11166.33)(0.05)}\left[1 - \frac{1}{(1+0.05)^8}\right]\right]}{\lg(1+0.05)}$$

$$K = \frac{\lg[(3.582197553)(0.323160638)]}{\lg(1.05)}$$

$$K = \frac{\lg(1.157625247)}{\lg(1.05} = 3$$

– El número de periodos diferidos son 3.

Rpta (21) $P_0 = 10,000$

1 . . . . 11 12 meses

0 $Ra = 1000$ 1000

$K = ?$

TEM = 0.02

Const: $$\boxed{K = \frac{\lg\left[\frac{Ra[(1+i)^n - 1]}{P i (1+i)^{n-1}}\right]}{\lg(1+i)}}$$

Reemplazando:

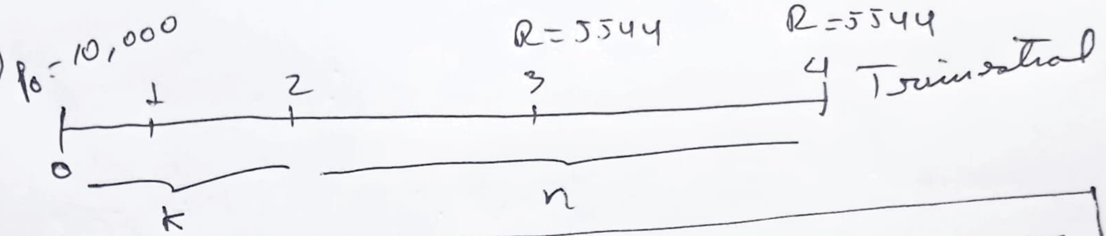
$$K = \frac{\lg\left[\frac{1000[(1+0.02)^{12} - 1]}{(10000)(0.02)(1+0.02)^{12-1}}\right]}{\lg(1+0.02)}$$

$$K = \frac{\lg(1.078684805)}{\lg(1.02)}$$

$$K = 3.824872552$$

– El número de periodos diferidos mensuales son $K = 3.824872552$.

Rpta 23



TET = ?

Como: $$\boxed{P_0 = R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]\left[\frac{1}{(1+i)^k}\right]}$$

Interpolamos:

	P_0	TET	
a	10033.29934	0.029	c
a, b	10000	TET	c, d
b	9999.317555	0.03	d

Como: $\frac{a}{b} = \frac{c}{d}$

Reemplazamos:

$$\frac{10033.29934 - 10000}{10000 - 9999.317555} = \frac{0.029 - TET}{TET - 0.03}$$

$$(48.79420923)(TET - 0.03) = 0.029 - TET$$

$$(49.79420923)TET - 1.463826277 = 0.029$$

$$(49.79420923)TET = 1.492826277$$

$$TET = 0.02997991734$$

– El TET cargado en el financiamiento es 2.998%